
AUTHENTICITY OF STABILITY OF STRATIFIED FLOWS OF MICROPOLAR FLUIDS MECHANICS CONCERNS

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ABSTRACT

Fluids are everywhere in nature. From a morning coffee to an evening bath, fluids are all around us. Water is a fluid and so is air. Fluids are either ideal or real depending upon how the viscosity changes in the various fluids. Also, fluids are “Newtonian” and “non-Newtonian” according to their response to an applied shear rate or shear stress. This paper of micropolar fluids has received considerable attention due to their applications in a number of processes that occur in industries such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, hematological suspensions, geomorphological sediments, exotic lubricants and colloidal and suspension solution.

Key Words : Fluids mechanics, solids, property

INTRODUCTION

Fluid mechanics concerns itself with the investigation of motion and equilibrium of fluids. We normally recognize three states of matter: solid, liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it flows under the action of the force. Its shape will change continuously as long as the force is applied. Eringen (1964) developed a continuum theory of micro fluid which, roughly speaking, is a fluent medium whose properties and behavior are affected by the local structure and micro motions of the constituent particles contained in each of its volume element. These fluids can support body moments, micro stress averages, stress moments and are influenced by the spin inertia which have no counterpart in the classical fluid theories. Due to the complexities of this theory, its potential has not thus far been exploited. Several sub-classes of this theory have been introduced by Eringen (1969, 1980). Micropolar fluid is one of them which has shown promise for predicting fluid behavior at micro scale. As mention in this paper, compared to the classical Newtonian fluids, micropolar fluids are characterized by two supplementary variables spin and micro-inertia, where the spin is responsible for the micro-rotations and the micro-inertia describes the distribution of atoms and molecules inside the fluid elements in addition to the velocity vector. Micropolar fluids can model ferrofluids, polymers, bubbly liquids, paints, liquid crystals with rigid molecules, clouds with dust, muddy fluids, colloidal fluids, blood and fluids containing certain additives.

REVIEW OF LITERATURE

Stavre (2003) determined an external field when the micro-rotation is equal to the vorticity of the fluid. He studied the discretization of the approximation and proved stability and convergence theorems. Murty (2001) investigated the combined effect of vertical throughflow and magnetic field on an electrically conducting micropolar fluid layer. Ibrahim et al. (2006) analyzed the non-classical heat conduction effects in Stokes' second problem of a micropolar fluid. Aziz (2006) studied the effect of radiation on magneto-hydrodynamic mixed convective steady laminar boundary layer flow of an optically thick electrically conducting viscous micropolar fluid past a moving semi-infinite vertical plate

for high temperature differences. Chen et al. (2011) investigated numerically the stability of a thin micropolar fluid flowing on a rotating circular disk. Ariman et al. (1973) presented an excellent review of the study of micropolar fluid and its applications. Batra (1978) considered a heat conducting compressible micropolar fluid at rest and filling a closed stationary rigid container. He showed that the energy of arbitrary disturbances of the fluid eventually decays. Perez-Garcia and Rubi (1979) presented a generalization to micropolar fluids of stability criteria developed by Glansdorff and Prigogine. Perdikis and Reptis (1996) analyzed the steady flow of a micropolar fluid past an unmoving plate by the presence of radiation. Hayakawa (2000) presented a systematic calculation of micropolar fluid flows around a sphere and a cylinder.

FORMULATION OF THE PROBLEM

Here we consider the stability of an incompressible micropolar fluid confined between infinite horizontal parallel planes at a finite distance d apart. In the Cartesian frame of reference, the axis of x is in the main flow direction and the axis of z is perpendicular to the planes.

The mathematical equations governing the motion of micropolar fluids for the considered flow are

$$\nabla \cdot \mathbf{q} = 0, \tag{1.1}$$

$$\rho \frac{D\mathbf{q}}{Dt} = -\nabla p + (\mu + \mu_r) \nabla^2 \mathbf{q} + \mu_r (\nabla \times \boldsymbol{\omega}) - \rho g \mathbf{e}_z, \tag{1.2}$$

$$\rho j \frac{D\boldsymbol{\omega}}{Dt} = (\varepsilon' + \beta'') \nabla (\nabla \cdot \boldsymbol{\omega}) + \gamma \nabla^2 \boldsymbol{\omega} + \mu_r (\nabla \times \mathbf{q}) - 2\mu_r \boldsymbol{\omega} \tag{1.3}$$

and $\frac{D\rho}{Dt} = 0$. (1.4)

BASIC STATE

In the undisturbed state, the fluid is at rest, therefore the basic state is characterized by

$$\begin{aligned} \mathbf{q} &= (0, 0, 0), \\ \boldsymbol{\omega} &= (0, 0, 0), \\ p &= p(z) \end{aligned}$$

ANALYTICAL DISCUSSION

Multiply equation (1.5) by w^* , the complex conjugate of w , integrate over the range of z and make use of boundary conditions, the real and imaginary parts of the resulting equation respectively yield

$$\begin{aligned} &\sigma_i \left[\int \left(2\sigma_r l + 2A - \frac{J_\mu k^2}{\sigma_r^2 + \sigma_i^2} \right) (|Dw|^2 + k^2 |w|^2) dz \right. \\ \text{and} & \left. + \int \{1 + l(1 + K)\} (|D^2 w|^2 + 2k^2 |Dw|^2 + k^4 |w|^2) dz - \int \left(\frac{2J_\mu k^2 A}{\sigma_r^2 + \sigma_i^2} \right) (|w|^2) dz \right] = 0 \end{aligned} \tag{1.5}$$

In the analysis given below, two cases have been discussed depending upon whether $J > 0$ or $J < 0$ i.e. whether the system is statically stable or statically unstable.

NUMERICAL DISCUSSION

CASE I: $J_\mu > 0$ (i.e. $D\rho_o < 0$)

For $J_\mu > 0$, we are able to find numerically the existence of oscillatory stable modes which were not found analytically.

In **Table - 1** values of the wave number k_c representing the change of mode from oscillatory stable to non-oscillatory stable are given. The table shows that though oscillatory stable modes convert into non-oscillatory stable modes for a very small difference (almost negligible) of wave number, but they exist. It is also clear that as $J_\mu > 0$ increases for a fixed K the oscillatory stable modes exist for large wave numbers and then jump to non-oscillatory stable modes and as K increases for a fixed J_μ , the range of oscillatory stable modes increases.

Also, we have calculated numerically the region of convergence for the semi-circular bounds given by inequality with centre (0, 0) and radius $I_1 = \sqrt{\frac{|J_\mu|(k^2 + 2A)}{2A + k^2 \{1 + l(1 + K)\}}}$. **Table - 2** shows that for fixed $A=0.5$, $l=1$ and $J_\mu = 0.25$, when K is increasing, the radius of the semi-circle representing unstable region is decreasing. This shows a stabilizing character of K (micropolar parameter).

Table 1: Critical wave numbers representing the change of modes from oscillatory stable to non-oscillatory stable

K	J	k_c (wave number) oscillatory stable modes	k_c (wave number) non-oscillatory stable modes
1	0.25	0.798078663	0.798078667
	0.50	0.942490731	0.942490733
	0.75	1.038212203	1.038212204
	1.00	1.111715737	1.111715738
	1.25	1.172159144	1.172159145
	1.50	1.223894570	1.223894571
	1.75	1.269360419	1.269360420
	2.00	1.310071283	1.310071284
3	0.25	0.638762746	0.638762749
	0.50	0.758041607	0.758041609
	0.75	0.836955184	0.836955185
	1.00	0.897394779	0.897394780
	1.25	0.946972220	0.946972221
	1.50	0.989311859	0.989311860
	1.75	1.026445604	1.026445605
	2.00	1.059635544	1.059635545

Table 2: Radius of semi-circle for different values of micropolar parameter

J_μ	k	I_1		
		K=1	K=3	K=5
0.25	1	.354	.289	.25
	2	.31	.244	.208

CONCLUSION

Stability of micropolar fluids between two parallel plates within the framework of linear analysis is examined. Two cases depending upon the statically stable ($J_\mu > 0$) or statically unstable density stratifications ($J_\mu < 0$) have been examined both analytically and numerically.

The main results obtained include

For $J_\mu > 0$

- the stability of non-oscillatory modes
- the circular bounds on the complex wave velocity of unstable modes
- the existence of oscillatory stable modes for large wave numbers
- the decrease in the range of oscillatory stable modes with the increase in the value of micropolar parameter K and the increase in the range of oscillatory stable modes with the increase in J_μ
- the stabilizing character of micropolar parameter K.

For $J_\mu < 0$

- the stability of oscillatory modes
- the bounds on the complex wave velocity of non-oscillatory unstable modes
- the stabilizing character of micropolar parameter K
- the increase in J_μ reduces the bounds of non-oscillatory unstable modes.

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